The one-particle momentum transfer in point-form spectator approximation

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The one-particle momentum transfer operator is derived in point-form spectator approximation for NN system. General expression is applied to elastic electron-deuteron scattering and to deuteron photodisintegration.

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I. INTRODUCTION

In Ref. [1] the elastic electron-deuteron scattering was described in frames of point-form (PF) of relativistic quantum mechanics (RKM). It was shown that in PF spectator approximation that the momentum of the unstruck particle (the spectator) is unchanged, while the impulse given to the struck particle is not the impulse given to the deuteron. In present paper this result is generalized and one-particle momentum transfer operator is derived for arbitrary NN-system following a general approach to construction of the electromagnetic current operator for relativistic composite system [2].

We define the momentum transfer Q_i^2 to the i-particle as an increment of i-particle 4-momentum q_i [1]

$$Q_i^2 = |(q_i' - q_i)^2|. \tag{1}$$

For interacting particles the individual 4-momenta are not defined before photon absorption (emission) as well as after it. Therefore we introduce operator Q_i^2 corresponding to the physical quantity of the momentum transfer Q_i^2.

The plan of the paper is as follows. In Sect. 2 we derive the one-particle momentum transfer operator. In Sect. 3 we show that our result is equivalent to the corresponding

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result of [1] for elastic electron-deuteron scattering. In Sect. 4 we consider deuteron photo-disintegration.

II. THE ONE-PARTICLE MOMENTUM TRANSFER OPERATOR

In the general case there are initial NN-state and associated initial s.c.m. (i.s.c.m.), and final NN-state and associated final s.c.m. (f.s.c.m.) Suppose that the photon momentum (momentum transfer) is along the z axis. Values of photon momentum and energy in i.s.c.m. are $|\mathbf{q}_\gamma|$ and q^0_γ correspondingly. Momentum transfer is $Q^2 = |\mathbf{q}_\gamma|^2 - (q^0_\gamma)^2$. Let P be the total 4-momentum of the NN-system, M be the mass of the NN-system, $G = P/M$ be the system 4-velocity. Index $i(f)$ means initial (final) state of the NN-system. Transformation from i.s.c.m. to the special system suggested by Lev [2] (L.s.) where

$$\mathbf{G}_f + \mathbf{G}_i = 0|_{L.s.} \tag{2}$$

is defined by angle $\Delta/2$ such that

$$\tanh \Delta/2 = h, \tag{3}$$

where $\mathbf{h} = \mathbf{G}_f/G^0_f|_{L.s.}$. The Lev frame (2) is not equivalent to the Breit frame defined by the condition $\mathbf{P}_f + \mathbf{P}_i = 0$ if $M_f \neq M_f$. In case of elastic electron-deuteron scattering these frames coincide.

From this point we may use a special derivation of Ref. [1] (Eqs. (4-8 of the present paper)).

The initial energies and z-components of momenta in L.s. are

$$\begin{aligned}
w_1 &= w \cosh \Delta/2 + q_z \sinh \Delta/2 \\
q_{1z} &= q_z \cosh \Delta/2 + w \sinh \Delta/2 \\
w_2 &= w \cosh \Delta/2 - q_z \sinh \Delta/2 \\
q_{2z} &= -q \cosh \Delta/2 + w \sinh \Delta/2,
\end{aligned} \tag{4}$$

where $\mathbf{q}$ and $w = \sqrt{\mathbf{q}^2 + m^2}$ are center of momentum variables, $\mathbf{q}$ is momentum of particle one (internal variable) After the photon absorption the z-component of the internal variable and corresponding energy change

$$q'_z = q_z \cosh \Delta \mp w \sinh \Delta \tag{5}$$

$$w' = w \cosh \Delta \mp q_z \sinh \Delta, \tag{6}$$

where the minus (plus) sign is used when particle one (two) is struck. The final energies and momenta in L.s. will then be

$$w_1' = w\cosh 3\Delta/2 - q_z \sinh 3\Delta/2$$
$$q_{1z}' = q_z 3\cosh \Delta/2 - w \sinh 3\Delta/2$$
$$w_2' = w_2; \qquad q_{2z}' = q_{2z}, \tag{7}$$

other components do not change. Some hyperbolic trigonometry reveals that

$$(q_1' - q_1)^2 = 4(q_z^2 - w^2)\sinh^2\Delta, \tag{8}$$

it follows from Eq. (3) that

$$\sinh\Delta = \frac{2h}{1-h^2}. \tag{9}$$

Since

$$q_z^2 - w^2 = -(m^2 + \mathbf{q}_\perp^2) = -(m^2 + \mathbf{q}^2 - \frac{(\mathbf{q}\cdot\mathbf{h})^2}{h^2}), \tag{10}$$

the momentum transfered to the struck particle is

$$Q_1^2 = -(q_1' - q_1)^2 = 16(m^2 + \mathbf{q}^2 - \frac{(\mathbf{q}\cdot\mathbf{h})^2}{h^2})\frac{h^2}{(1-h^2)^2}. \tag{11}$$

This is the general expression of the $Q_1^2 = Q_2^2$, in case of free two-particle states (for particles of equal masses). The parameter $\mathbf{h}$ does not depend on interaction and is specified by relative "position" of i.s.c.m. and f.s.c.m. In case of two interacting particles $\mathbf{q}$ and Q_i^2 are operators in the internal space. In impulse representation $\mathbf{q}$ is a variable of integration [1]. It is obvious that forcing on the plane wave this operator is equivalent to the multiplication by number $Q_1^2 = Q_2^2 > 0$ if $h \neq 0$.

It may be convenient to express $\sinh\Delta$ in Eq. (8) through the invariant masses of initial (M_i) and final (M_f) NN-states. With transition from i.s.c.m. to the special L.s. the 4-velocities of the initial and final states are transformed as

$$(1,0,0,0)|_{i.s.c.m.} \to (\frac{P_i^0}{M_i}, 0, 0, -|\mathbf{G}_i|)|_{L.s.}$$
$$(\frac{M_i + q_\gamma^0}{M_f}, 0, 0, \frac{|\mathbf{q}_\gamma|}{M_f})|_{i.s.c.m.} \to (\frac{P_f^0}{M_f}, 0, 0, |\mathbf{G}_f|)|_{L.s.}, \tag{12}$$

correspondingly, where order of components is (a^0, a^x, a^y, a^z). Lorentz transformations are linear, therefore sum of these 4-velocities transforms as

$$(\frac{M_f + M_i + q_\gamma^0}{M_f}, 0, 0, \frac{|\mathbf{q}_\gamma|}{M_f})|_{i.s.c.m.} \to (\frac{P_f^0}{M_f} + \frac{P_i^0}{M_i}, 0, 0, -|\mathbf{G}_i| + |\mathbf{G}_f|)|_{L.s.} \equiv$$
$$\equiv (\frac{P_f^0}{M_f} + \frac{P_i^0}{M_i}, 0, 0, 0)|_{L.s.}. \quad (13)$$

z-Component of this sum in L.s. is

$$\frac{|\mathbf{q}_\gamma|}{M_f} \cosh \Delta/2 - \frac{M_f + M_i + q_\gamma^0}{M_f} \sinh \Delta/2 = 0, \quad (14)$$

whence

$$\sinh \Delta/2 = \sqrt{\frac{|\mathbf{q}_\gamma|^2}{(M_f + M_i + q_\gamma^0)^2 - |\mathbf{q}_\gamma|^2}}, \quad (15)$$

and

$$\sinh^2 \Delta = 4 \sinh^2 \Delta/2 \cosh^2 \Delta/2 =$$
$$= 4 \frac{|\mathbf{q}_\gamma|^2}{(M_f + M_i + q_\gamma^0)^2 - |\mathbf{q}_\gamma|^2} (1 + \frac{|\mathbf{q}_\gamma|^2}{(M_f + M_i + q_\gamma^0)^2 - |\mathbf{q}_\gamma|^2}) =$$
$$= 4 \frac{|\mathbf{q}_\gamma|^2 (M_f + M_i + q_\gamma^0)^2}{((M_f + M_i + q_\gamma^0)^2 - |\mathbf{q}_\gamma|^2)^2}. \quad (16)$$

III. ELASTIC ELECTRON-DEUTERON SCATTERING

In case of elastic electron-deuteron scattering $M_i = M_f = m_D$ (m_D is mass of deuteron), $\frac{P_i^0}{M_i} = \frac{P_f^0}{M_f}|_{L.s.}$ therefore L.s. becomes the Breit system. Transformations (12) give for time components

$$\frac{P_i^0}{M_i}|_{L.s.} = \cosh \Delta/2 =$$
$$= \frac{P_f^0}{M_f}|_{L.s.} = \frac{m_D + q_\gamma^0}{m_D} \cosh \Delta/2 - \frac{|\mathbf{q}_\gamma|}{m_D} \sinh \Delta/2, \quad (17)$$

and for z-components of 4-velocities

$$|\mathbf{G}_i|_{L.s.} = -\sinh \Delta/2 =$$
$$= -|\mathbf{G}_i|_{L.s.} = -(-\frac{m_D + q_\gamma^0}{m_D} \sinh \Delta/2 + \frac{|\mathbf{q}_\gamma|}{m_D} \cosh \Delta/2), \quad (18)$$

where q_γ^0 and $\mathbf{q}_\gamma$ are energy and momentum of virtual photon correspondingly in the i.s.c.m. In this case i.s.c.m. coincides with laboratory system. Therefore

$$q_\gamma^0 \cosh \Delta/2 = |\mathbf{q}_\gamma| \sinh \Delta/2$$
$$(2m_D + q_\gamma^0) \sinh \Delta/2 = |\mathbf{q}_\gamma| \cosh \Delta/2. \quad (19)$$

Product of l.h.s.'s is equal to product of r.h.s.'s:

$$q_\gamma^0(2m_D + q_\gamma^0) = |\mathbf{q}_\gamma|^2, \tag{20}$$

i.e.

$$Q^2 = |\mathbf{q}_\gamma|^2 - (q_\gamma^0)^2 = 2m_D q_\gamma^0. \tag{21}$$

Exclusion of $|\mathbf{q}_\gamma|$ from Eqs. (19) gives

$$\tanh^2 \Delta/2 = \frac{q_\gamma^0}{2m_D + q_\gamma^0}, \tag{22}$$

Exclusion of q_γ^0 from last two gives

$$\tanh^2 \Delta/2 = \frac{Q^2}{Q^2 + 4m_D}, \tag{23}$$

finally

$$\sinh^2 \Delta/2 = \frac{Q^2}{4m_D^2}, \tag{24}$$

i.e. expression of [1] then deuteron absorbs virtual photon elastically, since.

$$\sinh \Delta = 2\sqrt{\frac{Q^2}{4m_D^2}}\sqrt{1 + \frac{Q^2}{4m_D^2}}. \tag{25}$$

Therefore expression inferred in Ref. [1] is a special case of Eq. (11).

IV. PHOTODISINTEGRATION OF DEUTERON

In case of deuteron photodisintegration the photon is real. In some cases (neglecting the final state interaction, or in some calculation schemes) we need to find action of operator (11) on free final state described by plain wave. For real photon $|\mathbf{q}_\gamma| = q_\gamma^0 = E_\gamma$. From (16) it follows that

$$\sinh^2 \Delta = 4\frac{E_\gamma^2(M_f + m_D + E_\gamma)^2}{((M_f + M_i + E_\gamma)^2 - E_\gamma^2)^2}, \tag{26}$$

where $M_i = m_D$, E_γ is photon energy in the i.s.c.m. In this case i.s.c.m. coincides with laboratory system. Since $M_f = \sqrt{(m_D + E_\gamma)^2 - E_\gamma^2}$ then

$$4\frac{(M_f + m_D + E_\gamma)^2}{((M_f + M_i + E_\gamma)^2 - E_\gamma^2)^2} = \frac{1}{(m_D + E_\gamma)^2 - E_\gamma^2}, \tag{27}$$

therefore

$$\sinh^2\Delta = \frac{E_\gamma^2}{(m_D+E_\gamma)^2-E_\gamma^2}, \tag{28}$$

this is a kinematical factor. Other factor in Eq. (10) contains operator $\mathbf{q}_\perp^2$. This operator acting on the plane wave in final state gives square of the proton momentum component in f.s.c.m. This component is orthogonal to $\mathbf{h}$ and therefore it does not change with transformations from i.s.c.m. to f.s.c.m. and to L.s. We find this quantity in i.s.c.m. using momentum conservation

$$\begin{aligned} E_\gamma^2 = |\mathbf{q}_n+\mathbf{q}_p|^2 = \\ = \mathbf{q}_n^2+\mathbf{q}_p^2+2(\pm\sqrt{w_n^2-m^2-\mathbf{q}_\perp^2}\sqrt{w_p^2-m^2-\mathbf{q}_\perp^2}-\mathbf{q}_\perp^2) = \\ = \mathbf{q}_n^2+\mathbf{q}_p^2+2(\pm\sqrt{w_n^2-x}\sqrt{w_p^2-x}-(x-m^2)) \end{aligned} \tag{29}$$

where $|\mathbf{q}_p|$ (w_p) and $|\mathbf{q}_n|$ (w_n) are momenta (energies) of final proton and neutron in i.s.c.m. (laboratory system for $\gamma d \to pn$). We take into account that $(\mathbf{q}_p)_\perp = -(\mathbf{q}_n)_\perp \equiv \mathbf{q}_\perp$. Independently on the relative direction of the parallel components of proton and neuteron momenta (i.e. on sign $\pm$) there is a single root

$$\begin{aligned} x = -(q_z^2-w^2) = m^2+\mathbf{q}_\perp^2 = \\ = \frac{(\mathbf{q}_n^2+\mathbf{q}_p^2+2m^2+2w_nw_p^2-E_\gamma^2)(\mathbf{q}_n^2+\mathbf{q}_p^2+2m^2-2w_nw_p^2-E_\gamma^2)}{4(\mathbf{q}_n^2+\mathbf{q}_p^2+2m^2-w_n^2-w_p^2-E_\gamma^2)} = \\ = -\frac{((w_n+w_p)^2-E_\gamma^2)((w_n-w_p)^2-E_\gamma^2)}{4E_\gamma^2} = \\ = -\frac{((m_D+E_\gamma)^2-E_\gamma^2)((w_n-w_p)^2-E_\gamma^2)}{4E_\gamma^2}. \end{aligned} \tag{30}$$

Inserting (28) and (30) into (8) we get action of the 4-momentum transfer operator in the internal space in case of plane wave in final state

$$Q_1^2 = -(q_1'-q_1)^2 = E_\gamma^2-(w_n-w_p)^2 = (2w_n-m_D)(2w_p-m_D). \tag{31}$$

Obviously $Q_2^2 = Q_1^2$. The operator may be substituted by this number only if the 3-vector parameter $\mathbf{h}$ is fixed and there is a plane wave in the bra part of the matrix element.

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